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Generation of two-mode entangled coherent states via a cavity QED system

Qing-Xia Mu, Yong-Hong Ma and Ling Zhou

School of physics and optoelectronic technology, Dalian University of Technology, Dalian 116024, People's Republic of China

E-mail: zhlhxn@dlut.edu.cn

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Abstract
A scheme is presented for the generation of entangled coherent states in a single-atom cavity-QED system. In the scheme, a three-level V-type atom interacts dispersively with a two-mode cavity and is driven by a classical field. We show that under large detuning conditions the cavity field can evolve into maximally entangled coherent states when one cavity mode is initially prepared in an odd coherent state. The effect of the cavity losses on the entanglement is also studied.

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(Some figures in this article are in colour only in the electronic version)

1. Introduction

Quantum superpositions of coherent states, the so-called Schrödinger cat states or entangled coherent states (ECSs), have attracted considerable attention in the recent literature [1–4]. Although coherent states are the closest quantum states to the classical depiction of a harmonic oscillator, because of the quantum interference between the coherent components, such superposition states can exhibit various nonclassical properties such as sub-Poissonian statistics, two-mode squeezing and violations of the Cauchy–Schwarz inequalities [5]. Numerous schemes have been proposed for generating these coherent superpositions. In [6], the nonlinear Mach–Zehnder interferometer is presented as a device whereby a pair of coherent states can be transformed into an entangled superposition of coherent states. Paternostro *et al* [7] show that ECSs can be produced via cross-phase modulation in a double electromagnetically induced transparency regime. The ion-trap systems have also been proposed to be a qualified candidate for the generation of such superposition states [8–10]. Experimentally, Schrödinger cat states have been generated in atomic systems using ion traps [11] and wave packets [12]. So far, in optical systems only Schrödinger kitten states

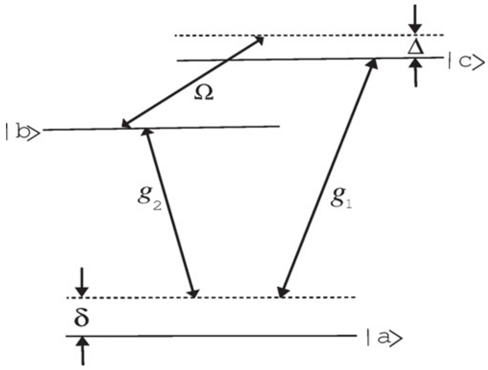

Figure 1. Schematic diagram of a three-level atom in V configuration. Two cavity modes interact with the transition $|c\rangle \leftrightarrow |a\rangle$ and $|b\rangle \leftrightarrow |a\rangle$, respectively, with the same detuning δ. Meanwhile, the external field with Rabi frequency Ω drives the transition $|c\rangle \leftrightarrow |b\rangle$ with detuning Δ.

with the form of a superposition of two coherent states with equal amplitude but different phases, such as $|\varphi\rangle = |\alpha\rangle + |\alpha\,\mathrm{e}^{\mathrm{i}\theta}\rangle$ with $|\alpha\rangle \ll 1$ have been produced in the laboratory [13].

Cavity quantum electrodynamics have been shown to be another promising environment for the preparation of coherent superpositions. Gerry [14] has proposed a method for generating Schrödinger cat states in a dispersive atom–cavity field interaction with a continuous external pump field. Solano *et al* [15] have presented a scheme to generate ECSs through the interaction of two cavity modes with a two-level atom. However, in their scheme the two cavity modes interact with the same atomic transition and thus will put restrictions on manipulation. In a recent paper, we have shown that a three-level Λ-type atom interacting with a two-mode field can entangle the two cavity modes under large detuning conditions [16]. Most recently, our group proposed that ECSs can be generated when considering a three-level V-type atom interacted with a doubly resonant cavity driven by classical fields [17]. But the atom detection is needed in these two schemes, and the decay of the excited level will destroy the entanglement in [17].

In this paper, we suggested a scheme to generate ECSs based on the off-resonant interaction of a three-level atom in V-type with a two-mode cavity driven by a classical field. We show that maximally ECSs can be generated by one cavity mode initially prepared in an odd coherent state. An analytical solution is also obtained when considering cavity decay. Compared with our recent research [16, 17], we do not need to detect the atom state. Compared with [15], the cavity modes in our scheme interact with different atomic transitions and thus can easily be manipulated.

2. The theoretical model and the generation of ECSs

To generate the desired effective interaction we will consider a three-level atom in V configuration trapped inside an optical cavity as depicted in figure 1. The atomic levels are $|a\rangle, |b\rangle$ and $|c\rangle$ with the corresponding energies ω_a, ω_b and ω_c. The allowed transitions $|c\rangle \leftrightarrow |a\rangle$ and $|b\rangle \leftrightarrow |a\rangle$ are excited off-resonantly by two cavity modes with the coupling constants g_1 and g_2, respectively, and with the same detuning δ. The external field with Rabi frequency Ω induces the dipole-forbidden transition $|c\rangle \leftrightarrow |b\rangle$ with detuning Δ. Ω can be realized by using a two-photon Raman transition via a fourth level [18].

Under the rotating wave approximation, the Hamiltonian for this system in the interaction picture can be written as ($\hbar = 1$)

$$H_I = g_1\left(a_1\sigma_{ca} + a_1^\dagger\sigma_{ac}\right) + g_2\left(a_2\sigma_{ba} + a_2^\dagger\sigma_{ab}\right) + \Omega\left(\sigma_{bc}\,\mathrm{e}^{\mathrm{i}\Delta t} + \sigma_{cb}\,\mathrm{e}^{-\mathrm{i}\Delta t}\right) + \delta(\sigma_{bb} + \sigma_{cc}), \tag{1}$$

where $\sigma_{ij} = |i\rangle\langle j\rangle(i, j = a, b, c)$ are atomic operators. $\delta = \omega_c - \omega_a - \omega_1 = \omega_b - \omega_a - \omega_2$, $\Delta = \omega_0 - (\omega_c - \omega_b)$; ω_1 and ω_2 are frequencies of the two cavity modes described respectively by the annihilation (creation) operators a_1 $\left(a_1^\dagger\right)$ and a_2 $\left(a_2^\dagger\right)$, and ω_0 is the frequency of the classical field. Considering the equation of motion for the atomic operators σ_{ca} and σ_{ba}

$$\begin{aligned} \mathrm{i}\frac{\mathrm{d}\sigma_{ca}}{\mathrm{d}t} &= g_1 a_1^\dagger(\sigma_{cc} - \sigma_{aa}) + g_2 a_2^\dagger\sigma_{cb} - \Omega\,\mathrm{e}^{\mathrm{i}\Delta t}\sigma_{ba} - \delta\sigma_{ca}, \\ \mathrm{i}\frac{\mathrm{d}\sigma_{ba}}{\mathrm{d}t} &= g_2 a_2^\dagger(\sigma_{bb} - \sigma_{aa}) + g_1 a_1^\dagger\sigma_{bc} - \Omega\,\mathrm{e}^{-\mathrm{i}\Delta t}\sigma_{ca} - \delta\sigma_{ba}, \end{aligned} \tag{2}$$

if the frequency detuning is sufficiently large, i.e., $\delta \gg g_1, g_2, \Omega, \Delta$, we can obtain the adiabatic solutions for σ_{ca} and σ_{ba} by setting $\mathrm{i}\frac{\mathrm{d}\sigma_{ca}}{\mathrm{d}t} = \mathrm{i}\frac{\mathrm{d}\sigma_{ba}}{\mathrm{d}t} = 0$ [19]. Substituting σ_{ca}, σ_{ba} and their Hermitian conjugates into Hamiltonian equation (1), we have

$$\begin{aligned} H_{\mathrm{eff}} = {}& \Omega\,(\sigma_{bc}\,\mathrm{e}^{\mathrm{i}\Delta t} + \sigma_{cb}\,\mathrm{e}^{-\mathrm{i}\Delta t}) + \delta(\sigma_{bb} + \sigma_{cc}) \\ &+ \frac{1}{\delta^2 - \Omega^2}\left\{\delta g_1^2\left(2a_1^\dagger a_1 + 1\right)(\sigma_{cc} - \sigma_{aa}) + \delta g_2^2\left(2a_2^\dagger a_2 + 1\right)(\sigma_{bb} - \sigma_{aa})\right. \\ &- \Omega g_1 g_2\left(a_1 a_2^\dagger\,\mathrm{e}^{\mathrm{i}\Delta t} + a_1^\dagger a_2\,\mathrm{e}^{-\mathrm{i}\Delta t}\right)(\sigma_{bb} + \sigma_{cc} - 2\sigma_{aa}) \\ &- \Omega\left[\left(g_1^2 a_1 a_1^\dagger + g_2^2 a_2^\dagger a_2\right)\mathrm{e}^{\mathrm{i}\Delta t}\sigma_{bc} + \left(g_1^2 a_1^\dagger a_1 + g_2^2 a_2 a_2^\dagger\right)\mathrm{e}^{-\mathrm{i}\Delta t}\sigma_{cb}\right] \\ &\left.+ 2\sigma g_1 g_2\left(a_1 a_2^\dagger\sigma_{cb} + a_1^\dagger a_2\sigma_{bc}\right)\right\}. \end{aligned} \tag{3}$$

If the initial state of the atom is prepared in the ground state $|a\rangle$, it will be confined in this state, and the cavity field is decoupled with the atomic part since only σ_{aa} will have action on $|a\rangle$ when we substitute equation (3) into $|\varphi(t)\rangle = \mathrm{e}^{-\mathrm{i}H_{\mathrm{eff}}t}|a\rangle$. In this case, the effective Hamiltonian describing the evolution of the cavity field is

$$H_a = -\frac{\delta}{\delta^2 - \Omega^2}\left[g_1^2\left(2a_1^\dagger a_1 + 1\right) + g_2^2\left(2a_2^\dagger a_2 + 1\right)\right] + \lambda\left(a_1 a_2^\dagger\mathrm{e}^{\mathrm{i}\Delta t} + a_1^\dagger a_2\,\mathrm{e}^{-\mathrm{i}\Delta t}\right), \tag{4}$$

where $\lambda = \frac{2g_1g_2\Omega}{\delta^2-\Omega^2}$. Next we choose $H_a^0 = -\frac{\delta}{\delta^2-\Omega^2}\left[g_1^2\left(2a_1^\dagger a_1 + 1\right) + g_2^2\left(2a_2^\dagger a_2 + 1\right)\right]$, $H_a^1 = \lambda\left(a_1 a_2^\dagger\,\mathrm{e}^{\mathrm{i}\Delta t} + a_1^\dagger a_2\,\mathrm{e}^{-\mathrm{i}\Delta t}\right)$, then perform the unitary transformation $U = \mathrm{e}^{-\mathrm{i}H_a^0 t}$ on H_a^1 and obtain

$$H_a^I = \lambda\left(a_1 a_2^\dagger\,\mathrm{e}^{\mathrm{i}(\Delta-\varepsilon)t} + a_1^\dagger a_2\,\mathrm{e}^{-\mathrm{i}(\Delta-\varepsilon)t}\right), \tag{5}$$

where $\varepsilon = \frac{2\delta(g_2^2-g_1^2)}{\delta^2-\Omega^2}$. With the choice of the parameter $\Delta = \varepsilon$ we have the evolution of the cavity field given by the Hamiltonian

$$H_a^I = \lambda\left(a_1 a_2^\dagger + a_1^\dagger a_2\right). \tag{6}$$

We recognize such a field Hamiltonian is a generator of $SU(2)$ coherent state if acting on the initial number state $|0, N\rangle$ [20]. However, the Hamiltonian cannot entangle the initial coherent state, since the time evolution of the system is

$$|\Phi(t)\rangle = \mathrm{e}^{-\mathrm{i}H_a^I t}|\Phi(0)\rangle = \mathrm{e}^{x_+K_+}\,\mathrm{e}^{K_0 ln x_0}\,\mathrm{e}^{x_-K_-}|\Phi(0)\rangle, \tag{7}$$

where $x_+ = x_- = -\tanh \mathrm{i}\lambda t, x_0 = (\cosh \mathrm{i}\lambda t)^{-2}$. These operators satisfy the $SU(2)$ commutation relations, i.e. $[K_0, K_+] = K_+, [K_0, K_-] = -K_-, [K_+, K_-] = 2K_0$, with $K_+ = a_1^\dagger a_2$, $K_- = a_1 a_2^\dagger$ and $K_0 = \frac{1}{2}\left[a_1^\dagger a_1 - a_2^\dagger a_2\right]$. If the initial state is $|\alpha_1, \alpha_2\rangle$, with $|\alpha_i\rangle = \mathrm{e}^{-|\alpha_i|^2/2}\sum_{n=0}^{\infty}\frac{(\alpha_i)^n}{\sqrt{n!}}|n\rangle$ being a coherent state of amplitude α_i, a direct calculation shows that

$$|\Phi(t)\rangle = \mathrm{e}^{x_+K_+}\,\mathrm{e}^{K_0 ln x_0}\,\mathrm{e}^{x_-K_-}|\alpha_1, \alpha_2\rangle = |\tilde{\alpha_1}, \tilde{\alpha_2}\rangle, \tag{8}$$

with $\tilde{\alpha_1} = \alpha_1 \cos\lambda t - \mathrm{i}\alpha_2 \sin\lambda t$, $\tilde{\alpha_2} = \alpha_2 \cos\lambda t - \mathrm{i}\alpha_1 \sin\lambda t$. Obviously, it is not an entangled state.

In order to generate ECS, we consider the cases that cavity mode 1 is initially prepared in the even or odd coherent states $|\phi_\pm(\alpha)\rangle = N_\pm(|\alpha\rangle \pm |-\alpha\rangle)$, with the normalization factors $N_\pm = \left(2 \pm 2\,\mathrm{e}^{-2|\alpha|^2}\right)^{-1/2}$. The even or odd coherent states $|\phi_\pm(\alpha)\rangle$ can be generated by sending Rydberg atoms passing through this two-mode cavity and interacting dispersively with cavity mode 1 (initially prepared in a coherent state $|\alpha\rangle$), as reported in [21]. By proper selection of the atomic velocity, one can produce a Schrödinger cat state in cavity mode 1 after the atom detection [21]. We assume cavity mode 2 is in the vacuum state $|0\rangle$, then the whole initial state for the cavity field can be expressed as

$$|\Psi(0)\rangle_\pm = N_\pm(|\alpha\rangle \pm |-\alpha\rangle) \otimes |0\rangle. \tag{9}$$

After an interaction time t, the state evolves into

$$|\Psi(t)\rangle_\pm = N_\pm(|\alpha\cos\lambda t, -\mathrm{i}\alpha\sin\lambda t\rangle \pm |-\alpha\cos\lambda t,\ \mathrm{i}\alpha\sin\lambda t\rangle). \tag{10}$$

In this way, we obtain a system of ECSs [6], which are themselves examples of two-mode cat states of even and odd type [5]. We now try to estimate the entanglement between two cavity modes. Although the entanglement criteria for the continuous variable system have been studied in [22, 23], they are still difficult to measure the entanglement of ECSs. Here, we quantify the entanglement of the two cavity modes by the familiar concept of concurrence [24], which has been generalized for judging ECSs by rebuilding two orthogonal and normalized states as a basis of the two-dimensional Hilbert space using original two coherent states [25, 26]. In our scheme, we can define the orthogonal and normalized basis as $|0\rangle = |\alpha\cos\lambda t\rangle$, $|1\rangle = (|-\alpha\cos\lambda t\rangle - P_1|\alpha\cos\lambda t\rangle)/M_1$ with $P_1 = \langle\alpha\cos\lambda t| - \alpha\cos\lambda t\rangle = \mathrm{e}^{-2|\alpha|^2\cos^2\lambda t}$, $M_1 = \sqrt{1-|P_1|^2}$ for cavity mode 1, and $|0\rangle = |-\mathrm{i}\alpha\sin\lambda t\rangle$, $|1\rangle = (|\mathrm{i}\alpha\sin\lambda t\rangle - P_2|-\mathrm{i}\alpha\sin\lambda t\rangle)/M_2$ with $P_2 = \langle-\mathrm{i}\alpha\sin\lambda t|\,\mathrm{i}\alpha\sin\lambda t\rangle = \mathrm{e}^{-2|\alpha|^2\sin^2\lambda t}$, $M_2 = \sqrt{1-|P_2|^2}$ for cavity mode 2. In this discrete basis, the concurrence of state equation (10) can be calculated as

$$C\pm = \frac{\sqrt{(1-\mathrm{e}^{-4|\alpha|^2\cos^2\lambda t})(1-\mathrm{e}^{-4|\alpha|^2\sin^2\lambda t})}}{1\pm\mathrm{e}^{-2|\alpha|^2}}, \tag{11}$$

where $+(-)$ means that cavity mode 1 is initially in the even (odd) coherent state. The average photon numbers of the two cavity modes can be easily derived as

$$\left\langle a_1^\dagger a_1\right\rangle_\pm = |\alpha|^2\cos^2(\lambda t)\frac{1\mp\mathrm{e}^{-2|\alpha|^2}}{1\pm\mathrm{e}^{-2|\alpha|^2}}, \qquad \left\langle a_2^\dagger a_2\right\rangle_\pm = |\alpha|^2\sin^2(\lambda t)\frac{1\mp\mathrm{e}^{-2|\alpha|^2}}{1\pm\mathrm{e}^{-2|\alpha|^2}}. \tag{12}$$

We plot the evolution of concurrence C_- (figure 2) and C_+ (figure 3) for different values of $|\alpha|$. Figure 2 shows that C_- periodically reaches the maximal value 1 at the evolution times $\lambda t = (2n+1)\pi/4 (n = 0, 1, \ldots)$ for any intensity values $|\alpha|$. Therefore, the maximally entangled coherent states of the two cavity modes can be prepared from the initial odd coherent state. Figure 3 shows that C_+ oscillates with time, and the maximal values increase with the increase of $|\alpha|$. One can clearly see it from the expression of equation (11). If cavity mode 1 is initially prepared in an odd coherent state, equation (11) shows that the two cavity modes evolve into the maximally ECSs when $\sin\lambda t = \cos\lambda t$, namely $\lambda t = (2n+1)\pi/4 (n = 0, 1, \ldots)$. Furthermore, when $|\alpha|$ is large enough, $\mathrm{e}^{-2|\alpha|^2} \approx 0$, and then equation (11) shows $C_+ = 1$ at time $\lambda t = (2n+1)\pi/4 (n = 0, 1, \ldots)$, so that we can approximately obtain the maximally ECSs if cavity mode 1 is initially prepared in the even coherent state. In fact, from numerical calculation, we find that when $|\alpha|$ is very large the dynamics of the entanglement between the two cavity modes are almost the same for C_- and C_+.

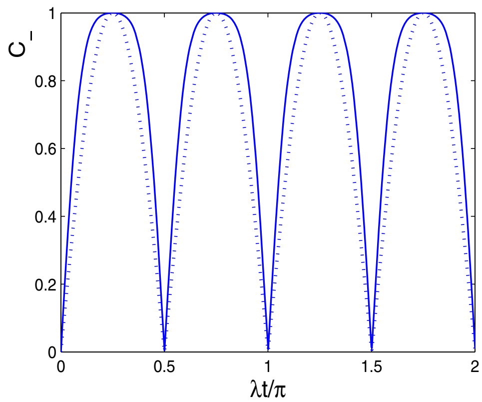

Figure 2. Concurrence C_- versus dimensionless time $\lambda t/\pi$ for (1)$|\alpha| = 0.5$ (dotted line), (2) $|\alpha| = 1.7$ (solid line).

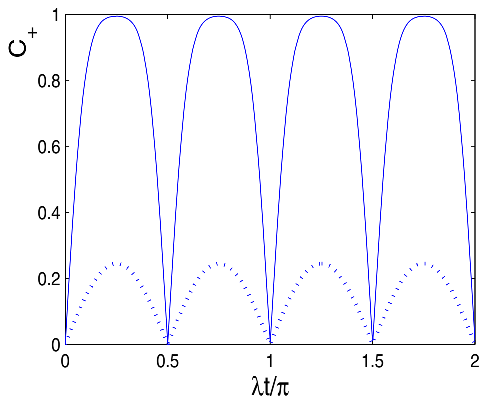

Figure 3. Concurrence C_+ versus dimensionless time $\lambda t/\pi$ for (1)$|\alpha| = 0.5$ (dotted line), (2) $|\alpha| = 1.7$ (solid line).

Next, we discuss the relation between the concurrence and the average photon numbers in two cavity modes. Comparing equation (11) and equation (12), we can clearly find that when the average photon numbers are equal, the concurrence reaches the maximum value. However, the concurrence will be zero as long as the average photon number in one of the cavity modes is zero. Take the case that cavity mode 1 is initially prepared in the even coherent state as an example; figure 4 clearly shows this point. In figure 4, the average photon numbers of two cavity modes oscillate with time and reach the maximal value alternately. When the photon number of one cavity mode reaches the maximal value, the photon number of the other cavity mode must be zero, then the concurrence vanishes. That is because it is the correlation between the two cavity modes that gives rise to the entanglement, if the average photon number in one of the modes vanishes, then the correlation no longer exists and the concurrence vanishes.

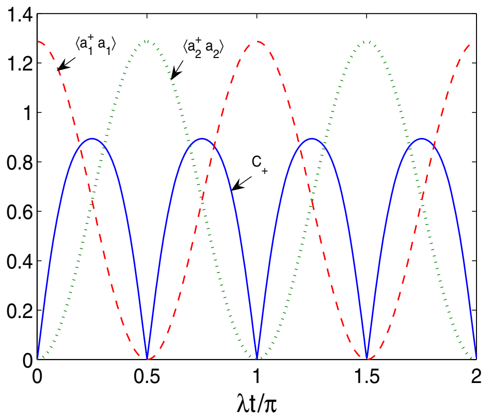

Figure 4. The time evolution of concurrence C_+ (solid line), average photon number $\langle a_1^\dagger a_1\rangle_+$ (dashed line) and $\langle a_2^\dagger a_2\rangle_+$ (dotted line) for the case that cavity mode 1 is initially prepared in the even coherent state, where $|\alpha| = 1.2$.

3. The effect of cavity decay on the entanglement between two cavity modes

In this section, the effect of the cavity losses on the entanglement is studied. Taking into account the dissipations of each mode in the vacuum environment that we obtain the motion equation of the density operator as

$$\dot{\rho} = -\mathrm{i}\lambda\left(a_1a_2^\dagger\rho - \rho a_1a_2^\dagger + a_1^\dagger a_2\rho - \rho a_1^\dagger a_2\right) + \frac{\kappa}{2}\sum_{i=1}^{2}\left(2a_i\rho a_i^\dagger - a_i^\dagger a_i\rho - \rho a_i^\dagger a_i\right). \quad (13)$$

For simplicity, we assume the two cavity modes have the same decay rate κ. Using the superoperator [27] technique and the $su(2)$ Lie algebra [28], the analytical solution of the system with the initial state equation (9) can be obtained as

$$\begin{aligned}\rho_\pm(t) = N_\pm^2\Big[&\big|\alpha\,\mathrm{e}^{-\frac{\kappa t}{2}}\cos\lambda t, -\mathrm{i}\alpha\,\mathrm{e}^{-\frac{\kappa t}{2}}\sin\lambda t\big\rangle\big\langle\alpha\,\mathrm{e}^{-\frac{\kappa t}{2}}\cos\lambda t, -\mathrm{i}\alpha\,\mathrm{e}^{-\frac{\kappa t}{2}}\sin\lambda t\big| \\ &+ \big|-\alpha\,\mathrm{e}^{-\frac{\kappa t}{2}}\cos\lambda t, \mathrm{i}\alpha\,\mathrm{e}^{-\frac{\kappa t}{2}}\sin\lambda t\big\rangle\big\langle-\alpha\,\mathrm{e}^{-\frac{\kappa t}{2}}\cos\lambda t, \mathrm{i}\alpha\,\mathrm{e}^{-\frac{\kappa t}{2}}\sin\lambda t\big| \\ &\pm \mathrm{e}^{-2|\alpha|^2(1-\mathrm{e}^{-\kappa t})}\Big(\big|\alpha\,\mathrm{e}^{-\frac{\kappa t}{2}}\cos\lambda t, -\mathrm{i}\alpha\,\mathrm{e}^{-\frac{\kappa t}{2}}\sin\lambda t\big\rangle\big\langle-\alpha\,\mathrm{e}^{-\frac{\kappa t}{2}}\cos\lambda t, \mathrm{i}\alpha\,\mathrm{e}^{-\frac{\kappa t}{2}}\sin\lambda t\big| \\ &+ \big|-\alpha\,\mathrm{e}^{-\frac{\kappa t}{2}}\cos\lambda t, \mathrm{i}\alpha\,\mathrm{e}^{-\frac{\kappa t}{2}}\sin\lambda t\big\rangle\big\langle\alpha\,\mathrm{e}^{-\frac{\kappa t}{2}}\cos\lambda t, -\mathrm{i}\alpha\,\mathrm{e}^{-\frac{\kappa t}{2}}\sin\lambda t\big|\Big)\Big]. \end{aligned} \quad (14)$$

When $k = 0$, which means that there are no cavity losses, we find the state equation (14) is exactly the same as equation (10). We still use the concurrence to measure the entanglement. The qubits for each cavity mode should be redefined as

$$\begin{aligned}|0\rangle &= \big|\alpha\,\mathrm{e}^{-\frac{\kappa t}{2}}\cos\lambda t\big\rangle, \\ |1\rangle &= \frac{\big|-\alpha\,\mathrm{e}^{-\frac{\kappa t}{2}}\cos\lambda t\big\rangle - P_1\big|\alpha\,\mathrm{e}^{-\frac{\kappa t}{2}}\cos\lambda t\big\rangle}{M_1}, \qquad \text{for cavity mode 1,} \\ |0\rangle &= \big|-\mathrm{i}\alpha\,\mathrm{e}^{-\frac{\kappa t}{2}}\sin\lambda t\big\rangle, \\ |1\rangle &= \frac{\big|\mathrm{i}\alpha\,\mathrm{e}^{-\frac{\kappa t}{2}}\sin\lambda t\big\rangle - P_2\big|-\mathrm{i}\alpha\,\mathrm{e}^{-\frac{\kappa t}{2}}\sin\lambda t\big\rangle}{M_2}, \qquad \text{for cavity mode 2,}\end{aligned} \quad (15)$$

with

$$P_1 = \big\langle\alpha\,\mathrm{e}^{-\frac{\kappa t}{2}}\cos\lambda t\big|-\alpha\,\mathrm{e}^{-\frac{\kappa t}{2}}\cos\lambda t\big\rangle = \exp\{-2|\alpha|^2\,\mathrm{e}^{-\kappa t}\cos^2\lambda t\}, \qquad M_1 = \sqrt{1-|P_1|^2},$$

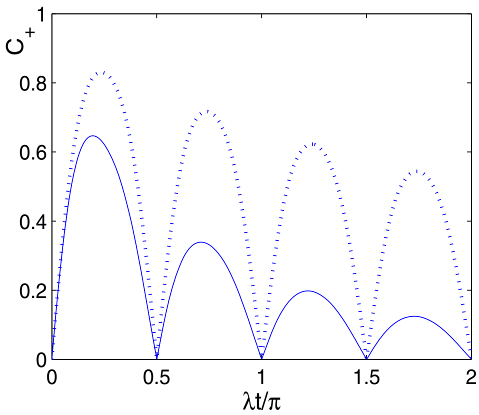

Figure 5. The time evolution of concurrence C_+ when considering cavity decay for $\kappa/\lambda = 0.1$ (dotted line) and $\kappa/\lambda = 0.5$ (solid line) when $|\alpha| = 1.2$.

$$P_2 = \left\langle -\mathrm{i}\alpha\, \mathrm{e}^{-\frac{\kappa t}{2}} \sin\lambda t \middle| \mathrm{i}\alpha\, \mathrm{e}^{-\frac{\kappa t}{2}} \sin\lambda t \right\rangle = \exp\{-2|\alpha|^2\, \mathrm{e}^{-\kappa t} \sin^2\lambda t\}, \qquad M_2 = \sqrt{1-|P_2|^2}. \tag{16}$$

Then the concurrence corresponding to equation (14) is found as

$$C_\pm = 2N_\pm^2 M_1 M_2\, \mathrm{e}^{-2|\alpha|^2(1-\,\mathrm{e}^{-\kappa t})}. \tag{17}$$

In figure 5, we plot the time evolution of the concurrence C_+ as an example in the presence of cavity losses. Not surprisingly, the amplitude of concurrence decreases with the increase of κ. The entanglement of the two cavity modes is gradually reduced by the effect of the environment. Therefore, a high-Q two-mode cavity is preferred in our scheme.

4. Conclusion

In conclusion, we have presented a scheme to generate ECSs by employing a three-level V-type atom with a two-mode field driven by an external laser field. It is shown that when the atom is prepared in the ground state $|a\rangle$, the two cavity modes can evolve into the ECSs. In particular, we find that maximally ECSs of two cavity modes can be generated if one cavity mode is initially prepared in an odd coherent state. The effect of cavity decay on the entanglement is also investigated by using concurrence.

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